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EXISTENCE OF POSITIVE SOLUTIONS OF HIGHER ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS

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ABSTRACT. The neutral differential equation

$$\frac{d^n}{dt^n}[x(t) + h(t)x(t-\tau)] + \sigma f(t, x(g(t))) = 0 \tag{1.1}$$

is considered under the following conditions: $n \geq 2$; $\sigma = \pm 1$; $\tau > 0$; $h \in C[t_0 - \tau, \infty)$; $g \in C[t_0, \infty)$, $\lim_{t\to\infty} g(t) = \infty$; $f \in C([t_0, \infty)\times(0, \infty))$, $f(t, u) \geq 0$ for $(t, u) \in [t_0, \infty)\times(0, \infty)$, and $f(t, u)$ is nondecreasing in $u \in (0, \infty)$ for each fixed $t \in [t_0, \infty)$. It is shown that, for the case where $h(t) > -1$ and $h(t) = h(t - \tau)$ on $[t_0, \infty)$, equation (1.1) has a positive solution $x(t)$ satisfying

$$x(t) = \left[\frac{c}{1 + h(t)} + o(1)\right] t^k \quad \text{as } t \to \infty$$

for some $c > 0$ if and only if

$$\int^{\infty} t^{n-k-1} f(t, a[g(t)]^k)\, dt < \infty \quad \text{for some } a > 0.$$

Here k is an integer with $0 \leq k \leq n - 1$.

1. Introduction. In this paper we consider the higher order neutral differential equation

$$\frac{d^n}{dt^n}[x(t) + h(t)x(t-\tau)] + \sigma f(t, x(g(t))) = 0, \tag{1.1}$$

where $n \geq 2$, $\sigma = \pm 1$ and $\tau > 0$, and the following conditions (i)–(iii) are assumed:

(i) $h : [t_0 - \tau, \infty) \to \mathbf{R}$ is continuous;

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