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DIFFEOMORPHISMS WITH THE AVERAGE-SHADOWING PROPERTY ON TWO-DIMENSIONAL CLOSED MANIFOLDS

KAZUHIRO SAKAI

ABSTRACT. The average pseudo-orbits and the average-shadowing property of diffeomorphisms on two-dimensional closed manifolds are considered, and the C^1 interior of the set of all diffeomorphisms satisfying the average-shadowing property is characterized as the set of all Anosov diffeomorphisms.

The notion of pseudo-orbits very often appears in several branches of the modern theory of dynamical systems, and, especially, the pseudo-orbit shadowing property usually plays an important role in the investigation of the stability theory. In [**1**] Blank introduced the notion of average pseudo-orbits as a certain generalization of the notion of pseudo-orbits (see also [**2**, p. 19]) and it was proved there that, for a certain kind of hyperbolic system f, every average pseudo-orbit of f is shadowed in average by some true orbit of f (the average-shadowing property).

Let M be a C^∞ closed manifold, that is, M is compact connected and $\partial M = \varnothing$, and let d be the distance induced from a Riemannian metric $\|\cdot\|$ on TM. Denote by Diff (M) the set of all diffeomorphisms on M endowed with C^1 topology. For $\delta > 0$, a sequence $\{x_i\}_{i=-\infty}^{\infty}$ of points in M is called a δ-*average pseudo-orbit* of $f \in$ Diff (M) if there is a number $N = N(\delta) > 0$ such that for all $n \geq N$, $k \in \mathbf{Z}$,

$$\frac{1}{n}\sum_{i=1}^{n} d(f(x_{i+k}), x_{i+k+1}) < \delta.$$

We say that f has the *average-shadowing property* if, for every $\varepsilon > 0$, there is a $\delta > 0$ such that every δ-average pseudo-orbit $\{x_i\}_{i=-\infty}^{\infty}$ is

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