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GALOIS REPRESENTATIONS ATTACHED TO THE PRODUCT OF TWO ELLIPTIC CURVES

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ABSTRACT. We study the images of mod p Galois representations attached to the abelian variety product of two elliptic curves. The case of two nonisogenous elliptic curves without complex multiplication has been considered by Serre [**3**]. In this paper we examine the case of two isogenous elliptic curves.

Let E_1, E_2 be two elliptic curves defined over a number field K. Let p be a prime number, and let $E_1[p]$ and $E_2[p]$ denote the group of p-torsion points of E_1 and E_2. The action of the absolute Galois group G_K of K on the p-torsion points of E_1 and E_2 defines the Galois representations

$$\rho_{E_1,p} : G_K \longrightarrow \mathrm{Aut}\,(E_1[p]), \quad \rho_{E_2,p} : G_K \longrightarrow \mathrm{Aut}\,(E_2[p])$$

and the homomorphism

$$\psi_p : G_K \longrightarrow \mathrm{Aut}\,(E_1[p]) \times \mathrm{Aut}\,(E_2[p]).$$

Let us denote

$$M_p := \{(s, s') \in \mathrm{Aut}\,(E_1[p]) \times \mathrm{Aut}\,(E_2[p]) : \det s = \det s'\}.$$

Let χ_p be the mod p cyclotomic character. We have that $\det \rho_{E_1,p} = \det \rho_{E_2,p} = \chi_p$, by the Weil pairing. Then the image $\psi_p(G_K)$ is contained in M_p.

Serre [**3**] studies the image $\psi_p(G_K)$ whenever the elliptic curves are without complex multiplication and not $\overline{K}$-isogenous. Using Falting's results [**2**] on the Tate conjecture, we have

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