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FOURIER-TYPE MINIMAL EXTENSIONS IN REAL L_1-SPACE

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ABSTRACT. Let $V \subset L_1([0,2\pi]^n)$ be a finite dimensional, shift-invariant subspace. Fix $w \in V$. In this paper we present an answer to the problem when the Fourier-type operator $F_w : L_1([0,2\pi]^n) \to V$ defined by

$$F_w(f) = f * w$$

is the only minimal extension of its restriction to V. Also the case of supremum norm will be considered and [**5**, p. 243] will be generalized.

0. Introduction. Let π_k denote the space of all trigonometric polynomials of degree less than or equal to k. Let $C_0(2\pi)$ denote the space of all continuous, real valued 2π-periodic functions. It is well known by the result of Lozinski [**10**] that the Fourier projection defined by

$$(F_k f)t = (f * D_k)t = (1/2\pi) \int_0^{2\pi} f(s) D_k(t-s)\, ds,$$

where $D_k t = \sum_{j=-k}^{k} e^{ijt}$, has the minimal norm among all the projections of $C_0(2\pi)$ onto π_k (see also [**1**], [**11**]). Also, it has been shown in [**3**] that F_k is the only projection from $C_0(2\pi)$ onto π_k of minimal norm. The problem of the unique minimality of the Fourier projection in more general context has been widely studied in literature (see, e.g., [**2**], [**5**], [**7**], [**8**], [**9**]).

In this paper we study the problem of the unique minimality of the Fourier-type extensions in the case of the L_1-norm. More precisely, for $u \in \mathbf{R}^n$ and $f \in L_1 = L_1([0,2\pi]^n)$, let

$$(I_u f)t = f(t+u). \tag{0.1}$$

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