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SHARP REGULARITY THEORY FOR ELASTIC AND THERMOELASTIC KIRCHOFF EQUATIONS WITH FREE BOUNDARY CONDITIONS

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ABSTRACT. We consider mixed problems for, initially, a two-dimensional model of an elastic Kirchoff equation with free boundary conditions (BC) and provide sharp trace and interior regularity results. The problem does not satisfy Lopatinski's conditions.

Pseudo-differential operator/micro-local analysis techniques are used. These results, in turn, yield a sharp regularity theory for the corresponding thermoelastic plate equation. The described sharp regularity theory, besides being of interest in itself, is critically needed in establishing a structural decomposition result of the corresponding thermoelastic semigroup with free BC [**12**], as well as in exact controllability problems.

1. Introduction and statement of main results.

Dynamical model. Let $\Omega \subset R^2$ be a bounded domain with smooth boundary Γ, say of class C^2. On Ω we consider the following two mixed (dual) problems for the so-called Kirchoff plate equation with free boundary conditions (BC) in the vertical displacement $w(t,\xi)$ or $u(t,\xi)$, $\xi = [\xi_1, \xi_2]$, respectively

(1.1a)
$$\mathcal{P}w \equiv w_{tt} - \gamma\Delta w_{tt} + \Delta^2 w = q, \qquad u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u = 0 \quad \text{in } Q,$$

(1.1b)
$$w(0,\cdot) = w_0,\ w_t(0,\cdot) = w_1, \quad u(T,\cdot) = 0,\ u_t(T,\cdot) = 0 \quad \text{in } \Omega,$$

(1.1c)
$$\mathcal{B}_1 w \equiv \Delta w + B_1 w = 0, \qquad \Delta u + B_1 u = g_1 \quad \text{in } \Sigma,$$

(1.1d)
$$\mathcal{B}_2 w \equiv \frac{\partial \Delta w}{\partial \nu} + B_2 w - \gamma \frac{\partial w_{tt}}{\partial \nu} \equiv 0; \qquad \frac{\partial \Delta u}{\partial \nu} + B_2 u - \gamma \frac{\partial u_{tt}}{\partial \nu} = g_2 \quad \text{in } \Sigma;$$

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