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UNIVERSAL BINARY POSITIVE DEFINITE HERMITIAN LATTICES

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ABSTRACT. We will determine all universal integral lattices on binary positive definite Hermitian spaces over arbitrary imaginary quadratic fields, where a positive definite lattice is said to be 'universal' if it represents all positive rational integers. A.G. Earnest and A. Khosravani determined universal binary Hermitian lattices when the imaginary quadratic fields have class number 1. In this paper we will extend the result to the case of fields with arbitrary class numbers and obtain nine new universal binary Hermitian lattices up to equivalence, including nonfree lattices.

1. Introduction. Lagrange [**5**] proved that any positive rational integer is a sum of four squares. In other words, the quaternary quadratic form $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2$ represents all positive rational integers. We call such a positive definite form a *universal* form. Ramanujan [**7**] showed that there are 55 universal diagonal quaternary quadratic forms in all. More generally, we know that there are only a finite number of universal integral forms with cross products but have not yet determined them completely, when they have an odd cross product. Recently, Earnest and Khosravani [**2**] investigated the similar problem for binary classic integral Hermitian forms over the imaginary quadratic fields of class number one and determined the classes of universal forms in the case. We say a lattice or a form is *classic integral* if its scale ideal is integral. As for the representation of numbers, binary Hermitian forms over quadratic fields can be regarded as quaternary quadratic forms over the rational number field. Accordingly, universal binary Hermitian forms provide universal quaternary quadratic forms, which may not be classic integral whereas Hermitian forms are classic. In this paper we will determine all universal binary Hermitian lattices for arbitrary imaginary quadratic fields.

In Section 2 we will give a correspondence of Hermitian lattices to quadratic lattices. In Section 3 we will obtain universal and potentially

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