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QUASINORMAL OPERATORS SIMILAR TO IRREDUCIBLE ONES

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ABSTRACT. We show that a quasinormal operator T on an infinite-dimensional Hilbert space is similar to an irreducible operator if and only if T is not quadratic and $T - \lambda I$ is not finite-rank for any $\lambda \in \mathbf{C}$.

1. Introduction. Throughout this paper, all operators are bounded and linear on complex Hilbert spaces. An operator T is said to be quasinormal if T commutes with T^*T. An operator is said to be irreducible if it commutes with no projection other than 0 and I, and is said to be reducible otherwise. The aim of this paper is to obtain necessary and sufficient conditions for a quasinormal operator to be similar to an irreducible operator.

Every operator T on a nonseparable Hilbert space is reducible. However, there are some operators on a separable Hilbert space which are reducible but are similar to irreducible ones. From now on, we only have to consider separable Hilbert space operators. Gilfeather [**5**] proved that every normal operator without eigenvalue is similar to an irreducible operator. Later on, Fong and Jiang [**4**] improved Gilfeather's work by allowing the presence of eigenvalues. In this paper we extend Fong and Jiang's result to quasinormal operators as follows.

Main Theorem. *A quasinormal operator T on an infinite-dimensional Hilbert space is similar to an irreducible operator if and only if T is not quadratic and $T - \lambda I$ is not finite-rank for any $\lambda \in \mathbf{C}$.*

We provide a similar theorem on finite-dimensional Hilbert spaces in [**7**].

Gilfeather [**5**] used binormal operators (defined in [**2**]) to prove that every quadratic operator T is always reducible. Here we give a much

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