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SOME RESULTS ON MEAN LIPSCHITZ SPACES OF ANALYTIC FUNCTIONS

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ABSTRACT. If f is a function which is analytic in the unit disk Δ and has a nontangential limit $f(e^{i\theta})$ at almost every $e^{i\theta} \in \partial\Delta$ and $1 \leq p \leq \infty$, then $\omega_p(\cdot, f)$ denotes the integral modulus of continuity of order p of the boundary values $f(e^{i\theta})$ of f. If $\omega : [0, \pi] \to [0, \infty)$ is a continuous and increasing function with $\omega(0) = 0$ and $\omega(t) > 0$ if $t > 0$ then, for $1 \leq p \leq \infty$, the mean Lipschitz space $\Lambda(p, \omega)$ consists of those functions f which belong to the classical Hardy space H^p and satisfy $\omega_p(\delta, f) = O(\omega(\delta))$ as $\delta \to 0$. If, in addition, ω satisfies the so-called Dini condition and the condition b_1, we say that ω is an admissible weight. If $0 < \alpha \leq 1$ and $\omega(\delta) = \delta^\alpha$, we shall write Λ_α^p instead of $\Lambda(p, \omega)$, that is, we set $\Lambda_\alpha^p = \Lambda(p, \delta^\alpha)$.

In this paper we obtain several results about the Taylor coefficients and the radial variation of the elements of the spaces $\Lambda(p, \omega)$. In particular, if ω is an admissible weight, then we give a complete characterization of the power series with Hadamard gaps which belong to $\Lambda(p, \omega)$.

If f is an analytic function in Δ and $\theta \in [-\pi, \pi)$, we let $V(f, \theta)$ denote the radial variation of f along the radius $[0, e^{i\theta})$. We also define the exceptional set $E(f)$ associated to f as $E(f) = \{e^{i\theta} \in \mathbf{T} : V(f, \theta) = \infty\}$. For any given $p \in [1, \infty]$, we obtain a characterization of those admissible weights ω for which the implication

$$f \in \Lambda(p, \omega) \Longrightarrow E(f) = \varnothing,$$

holds. We also obtain a number of results about the "size" of the exceptional set $E(f)$ for $f \in \Lambda_\alpha^p$.

1. Introduction. Let Δ denote the unit disk $\{z \in \mathbf{C} : |z| < 1\}$ and $\mathbf{T}$ the unit circle $\{\xi \in \mathbf{C} : |\xi| = 1\}$. If $0 < r < 1$ and g is a function

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