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SINGULAR POINTS FOR TILINGS OF NORMED SPACES

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ABSTRACT. A point x in a normed space X is said to be singular for a given tiling of X whenever each neighborhood of x intersects infinitely many tiles. We show that, when X is infinite-dimensional and all tiles are convex, special points in the boundary of tiles (like extreme points or PC points, if any) must be singular. Under the further assumptions that X is separable and doesn't contain c_0, singular points abound among the smooth points of any bounded tile. Finally, in any normed space a tiling is constructed which is free of singular points and whose members are both bounded and star-shaped; this disproves the conjecture that Corson's theorem might apply to star-shaped bounded coverings.

Introduction. Throughout this paper, X denotes a normed space over the reals.

A collection τ of subsets of X is a *covering* of X whenever each element of X belongs to some member of τ. If n is a cardinal number, a point x of X is said to be *n-singular* for τ if each neighborhood of x meets at least n different members of τ. For simplicity, $\aleph_0$-singular points will be called *singular points*. We say that τ is *locally finite* at x provided x is not a singular point for τ, and that τ is *locally finite* when it is locally finite at each point of X. A subset of X is a *body* if it is different from X itself and is the closure of its nonempty interior. A covering of X by bodies is called a *tiling* of X whenever any two different members of it have disjoint interiors. The elements of such a covering are called *tiles*. When adjectives (like "bounded," "convex," "star-shaped," etc.) are applied to a collection τ of subsets of X, it means that they apply to each member of τ.

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