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EQUAL SUMS OF SEVENTH POWERS

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ABSTRACT. Until now only three numerical solutions of the diophantine equation $x_1^7 + x_2^7 + x_3^7 + x_4^7 = y_1^7 + y_2^7 + y_3^7 + y_4^7$ are known. This paper provides three numerical solutions in positive integers of the hitherto unsolved system of simultaneous diophantine equations $x_1^k + x_2^k + x_3^k + x_4^k = y_1^k + y_2^k + y_3^k + y_4^k$, $k = 1, 3$ and 7.

Parametric solutions of the diophantine equation

$$\sum_{i=1}^{n} x_i^7 = \sum_{i=1}^{n} y_i^7 \tag{1}$$

have been given by Sastri and Rai [**5**] when $n = 6$ and by Gloden [**3**], [**4**] when $n = 5$. When $n = 4$, only three numerical solutions of (1) are known. These were discovered by Ekl [**1**], [**2**] via computer search.

In this paper we obtain three numerical solutions in positive integers of the hitherto unsolved system of diophantine equations

$$\sum_{i=1}^{4} x_i^k = \sum_{i=1}^{4} y_i^k, \quad k = 1, 3, 7. \tag{2}$$

To solve the system of equations (2), we write

$$\begin{aligned} x_1 &= X_1 - X_2 - X_3, & y_1 &= Y_1 - Y_2 - Y_3,\\ x_2 &= -X_1 + X_2 - X_3, & y_2 &= -Y_1 + Y_2 - Y_3,\\ x_3 &= -X_1 - X_2 + X_3, & y_3 &= -Y_1 - Y_2 + Y_3,\\ x_4 &= X_1 + X_2 + X_3, & y_4 &= Y_1 + Y_2 + Y_3. \end{aligned} \tag{3}$$

Then we have the identities

$$\sum_{i=1}^{4} x_i = 0, \qquad \sum_{i=1}^{4} x_i^3 = 24 X_1 X_2 X_3,$$

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