ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 3, Fall 2000

ON EQUAL SUMS OF SIXTH POWERS

AJAI CHOUDHRY

ABSTRACT. This paper provides a method of generating infinitely many integer solutions of the simultaneous equations $a^r + b^r + c^r = d^r + e^r + f^r$ where $r = 1, 2$ and 6. Several numerical solutions of this system of equations have also been obtained in this paper.

This paper deals with the simultaneous diophantine equations given by

$$a^r + b^r + c^r = d^r + e^r + f^r \tag{1}$$

where $r = 1, 2$ and 6. Numerical and parametric solutions of (1) with $r = 2$ and 6 have been obtained earlier by Subba Rao [**9**], Brudno [**2, 3**], Bremner [**1**], Choudhry [**4**] and Delorme [**5**]. It has been noted by Guy [**6**, p. 142] that all the known simultaneous solutions of (1) with $r = 2$ and 6 also satisfy (with appropriately chosen signs) the following three equations

$$\begin{aligned} a^2 + ad - d^2 &= f^2 + fc - c^2 \\ b^2 + be - e^2 &= d^2 + da - a^2 \\ c^2 + cf - f^2 &= e^2 + eb - b^2. \end{aligned} \tag{2}$$

Guy has asked the question whether there exists a counterexample which, while satisfying (1) for $r = 2$ and 6, does not satisfy the three equations given by (2). We also note that there exist solutions of (1) with $r = 6$ and $r \neq 2$. Lander, Parkin and Selfridge [**7**] gave one such numerical solution while Montgomery (as quoted by Guy [**6**, p. 142]) has listed 18 such solutions.

We will first obtain a numerical solution of (1) with $r = 1, 2$ and 6. This solution does not satisfy the three equations given by (2) and thus provides a counterexample asked for by Guy. Next we will use the

Received by the editors on January 11, 1999, and in revised form on February 18, 1999.