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VECTOR BUNDLES ON A FORMAL NEIGHBORHOOD OF A CURVE IN A SURFACE

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ABSTRACT. Here we study vector bundles in a formal or tubular neighborhood of a smooth projective curve C in a complex surface W. In several cases (e.g., if C has genus 0 and its normal bundle has degree -1 or -2) we attach to every such bundle a series of discrete invariants and "simpler" bundles, and we study the set of all bundles with fixed invariants.

0. Introduction. Let W be either a smooth connected quasi-projective surface defined over an algebraically closed field $\mathbf{K}$ or a smooth connected two-dimensional complex manifold. Let $C \subset W$ be a smooth connected curve of genus $q \geq 0$ and U either the formal completion of W along C or, in the complex analytic case, a small tubular neighborhood of C in W for the Euclidean topology. We want to study algebraic (or complex analytic) vector bundles on U. Even more, we want to study families of vector bundles on U "parametrized" (not one-to-one and usually not even generically finite to one) by integral varieties or irreducible and reduced complex spaces. In some cases a natural topological structure appears which allows us to say that a family of vector bundles is in the closure of another set of vector bundles. We give an easy example. Let $\pi : W \to \mathbf{P}^2$ be the blowing-up of the complex plane at a point. Let U be a small open Euclidean neighborhood of the exceptional divisor C on W. Consider a rank two holomorphic bundle E over W with $E \mid C \cong \mathbf{O}_C(2) \oplus \mathbf{O}_C(-2)$. A simple application of [**3**, Theorem 2.1] tells us that $E \mid U$ can be given by a 2×2 transition matrix of the form (g_{ij}) with $g_{11} = z^2$, $g_{22} = z^{-2}$, $g_{21} = 0$ and $g_{12} \in \mathbf{C}[z, u]$ with g_{12} of the form $g_{12} = (p_{10} + p_{11}z)u + p_{21}zu^2$

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