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ON FUNCTIONAL REPRESENTATION OF COMMUTATIVE LOCALLY A-CONVEX ALGEBRAS

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ABSTRACT. We shall give a Gelfand type of representation of commutative locally A-convex algebras by using a certain family of seminorms defined on the carrier space of this algebra. By using this representation we give a generalization of locally convex uniform algebras.

1. Introduction. Let (A, T) be a commutative algebra over the complex numbers equipped with a topology T. If A has unit element it will be denoted by e. In this paper we assume that the topology T on A has been given by means of a family $\mathcal{P} = \{p_\lambda \mid \lambda \in \Lambda\}$ of seminorms on A. This topology will be denoted by $T(\mathcal{P})$. We assume that $T(\mathcal{P})$ is a Hausdorff topology (i.e., from the condition $p_\lambda(x) = 0$, $x \in A$, for all $\lambda \in \Lambda$ it follows that $x = 0$). Suppose further that $\mathcal{P}$ has the following property. If λ and $\mu \in \Lambda$ then $\max\{p_\lambda, p_\mu\} \in \mathcal{P}$, i.e., $\mathcal{P}$ is directed. This property is needed in some place, but it is not necessary in general. We shall say that $(A, T(\mathcal{P}))$ is a locally A-convex algebra if for each $x \in A$ and $\lambda \in \Lambda$ there is some constant $M_{(x,\lambda)} > 0$ (depending on x and λ) such that

$$(1) \qquad p_\lambda(xy) \leq M_{(x,y)} p_\lambda(y) \quad \text{for all } y \in A.$$

If the above $M_{(x,\lambda)}$ does not depend on λ, i.e., (1) holds for all $\lambda \in \Lambda$ for some constant $M_x > 0$ depending only on x, then we say that $(A, T(\mathcal{P}))$ is a locally uniformly A-convex algebra. Furthermore, we say that $(A, T(\mathcal{P}))$ is locally m-convex if $p_\lambda(xy) \leq p_\lambda(x)p_\lambda(y)$ for all x and $y \in A$ and $\lambda \in \Lambda$. Obviously a locally m-convex algebra is locally A-convex. Note that the multiplication in locally A-convex algebra is in general only separately continuous and in locally m-convex algebra jointly continuous. The concepts of A-convex and uniformly A-convex algebras were introduced in [**13**], [**14**] and [**15**]. See also [**9**], [**21**], [**22**], [**23**] and [**24**]. A standard example of uniformly locally A-convex algebra is an algebra of bounded continuous complex-valued functions

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