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UPPER AND LOWER SOLUTIONS METHOD FOR EVEN ORDER TWO POINT BOUNDARY VALUE PROBLEMS

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ABSTRACT. The note shows the existence of solutions to an even order boundary value problem for ordinary differential equation with boundary conditions involving even order derivatives in the case when upper and lower solutions of the problem are known.

1. We will be concerned here with the existence of solutions to the following boundary value problem (BVP for short)

$$u^{(2k)} = f(t, u, u'', \dots, u^{(2k-2)}), \tag{1.1}$$

$$u^{(2j-2)}(0) = 0, \qquad u^{(2j-2)}(1) = 0, \quad j = 1, \dots, k, \tag{1.2}$$

where $f : [0,1] \times \mathbf{R}^k \to \mathbf{R}$ is continuous, in the case when upper/lower solutions corresponding to the problem are assumed to exist.

In contrast to a broad literature dealing with upper and lower solution methods applied to second order BVP's (see, e.g., [**5**] for the extensive literature on periodic BVP's), the number of papers devoted to BVP for the higher order differential equations is rather small. For more recent publications, see, e.g., [**6**], [**4**], [**7**] considering two point fourth order BVP's or paper [**1**] studying periodic problems. Further references can be found in the quoted papers.

In [**1**], [**4**], [**7**] the existence of solutions to BVP's is shown by finding two monotone sequences of functions converging uniformly to solutions of BVP's considered. The approach used in [**6**] is different, the fourth order differential equations as well as the systems of two second order equations together with various kinds of nonlinear boundary conditions are replaced by BVP's for quasilinear equations.

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