ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 4, Winter 2001

SOME OF THE PROPERTIES OF THE SEQUENCE OF POWERS OF PRIME NUMBERS

LAURENŢIU PANAITOPOL

ABSTRACT. The study of the increasing sequence $(q_n)_{n\geq 1}$ of natural numbers that are powers of prime numbers (i.e., the numbers of the form p^α, for every prime number p and every integer $\alpha \geq 1$) shows us that there is a perfect similarity between this one and the sequence $(p_n)_{n\geq 1}$ of prime numbers. The Landau theorem (see [**3**]) and the Šcherk theorem ([**6**]) have an equivalent for the numbers q_n. We can show that the sequence $(q_n)_{n\geq 1}$ is neither convex nor concave by using the classical results on the distribution of primes.

1. Introduction. Let $\pi^*(x)$ denote the number of all powers of primes not exceeding x, i.e.,

$$\pi^*(x) = \text{card}\,\{\text{there exist } \mid p \text{ prime and } \alpha \geq 1 \text{ integers such that } n = p^\alpha \leq x\}. \tag{1}$$

The definition of Mangold's function Λ and Chebyshev's function Ψ deals with these numbers.

Let $(q_n)_{n\geq 1}$ be the sequence of these numbers: $q_1 = 2$, $q_2 = 3$, $q_3 = 4$, $q_4 = 5$, $q_5 = 7$, $q_6 = 8$, $q_7 = 9\dots$. It is obvious that the only sequence of four consecutive numbers belonging to $(q_n)_{n\geq 1}$ is 2, 3, 4, 5.

Triples of consecutive numbers which are included in the $(q_n)_{n\geq 1}$ are 2, 3, 4; 3, 4, 5; 7, 8, 9. Indeed, for $n > 2$ such a triple is given by $q_{n-1} = 2^k-1$, $q_n = 2^k$, $q^{n+1} = 2^k+1$ and, because one of these numbers is a multiple of three, it is obvious that $2^k - 1 = 3^n$ or $2^k + 1 = 3^n$. The solutions of these equations are (2,1) or (3,2), and the assertion is justified.

1991 AMS *Mathematics Subject Classification.* 11A25, 11N05.
Key words and phrases. Prime numbers, powers of primes, convex or concave sequence, Landau's theorem, Scherk's theorem.
Received by the editors on February 11, 2000, and in revised form on September 7, 2000.