

**THE CARTESIAN CLOSED TOPOLOGICAL  
HULL OF THE CATEGORY OF  
APPROACH UNIFORM SPACES**

MARK NAUWELAERTS

**ABSTRACT.** The category **AUnif** of approach uniform spaces and uniform contractions properly combines uniform spaces and extended pseudo-metric spaces but (like **Unif**) lacks convenience, such as cartesian closedness. This paper therefore considers its cartesian closed topological hull, which is first described as a subcategory of **SAULim**, the category of semi-approach uniform limit spaces and uniform contractions. This hull is then also given a description inside the topological universe hull of **AUnif** and is shown to be a reasonable generalization of the corresponding hull of **Unif**. Furthermore, some referencing notes are provided with respect to similar results that can be obtained when starting from **qAUnif** (where symmetry assumptions are omitted).

**1. Introduction.** It is often desirable and useful for a (concrete) category to have extra properties in addition to just being nicely topological, such as being cartesian closed topological (CCT). However, many categories are not cartesian closed, which has inspired a theory of CCT extensions of such (failing) categories, where the least such CCT extension of a given concrete category, the CCT hull of a category, is especially interesting.

For instance, in [2], Adámek and Reiterman constructed the CCT hull of **Unif**, the category of uniform spaces (and uniformly continuous maps), and in [3], they described the CCT hull of the category  $(p)\mathbf{MET}^{(\infty)}$  of (extended pseudo-)metric spaces (and nonexpansive maps). Later the author added to these results by describing the CCT hull of the category **qUnif** of quasi-uniform spaces (and uniformly continuous maps) [16] (and thereby also adding to an alternative characterization of the CCT hull of **Unif** by Alderton and Schwarz [4]) and by describing the CCT hull of the category  $pq\mathbf{MET}^{\infty}$  of extended

---

Received by the editors on May 18, 2000.  
Author is the Research Assistant of the Fund for Scientific Research – Flanders (Belgium) (F.W.O.).