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HARMONIC BESOV SPACES ON THE UNIT BALL IN $\mathbf{R}^n$

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ABSTRACT. We define and characterize the harmonic Besov space B^p, $1 \le p \le \infty$, on the unit ball B in $\mathbf{R}^n$. We prove that the Besov spaces B^p, $1 \le p \le \infty$, are natural quotient spaces of certain L^p spaces. The dual of B^p, $1 \le p < \infty$, can be identified with B^q, $1/p + 1/q = 1$, and the dual of the little harmonic Bloch space B_0 is B^1.

1. Introduction. Let $d\nu$ be the volume measure on the unit ball $B = B_n$ in R^n normalized so that B has volume equal to one. For any real $\alpha > 0$ we consider the measure $d\nu_\alpha(x) = c_\alpha(1-|x|^2)^{\alpha-1}\,d\nu(x)$ where the constant c_α is chosen so that $d\nu_\alpha$ has total mass 1. An integration in polar coordinates shows that $c_\alpha = (2/n)[B(n/2,\alpha)]^{-1}$. See [**1**]. Also, we let $d\tau(x) = (1-|x|^2)^{-n}\,d\nu(x)$.

For f harmonic on B, $f \in h(B)$, and any positive integer m, we write $|\partial^m f(x)| = \sum_{|\alpha|=m} |\partial^\alpha f(x)|$, where $\partial^\alpha f(x) = (\partial^{|\alpha|} f/\partial x^\alpha)(x)$, α a multi-index.

For $1 \le p \le \infty$, the harmonic Besov space $B^p = B^p(B)$ consists of harmonic functions f on B such that the function $(1-|x|^2)^k|\partial^k f(x)|$ belongs to $L^p(B, d\tau)$ for some positive integer $k > (n-1)/p$. We note that the definition is independent of k (see Theorem 3.2).

Let B_0 be the subspace of B^∞ consisting of functions $f \in h(B)$ with

$$(1-|x|^2)^k|\partial^k f(x)| \longrightarrow 0, \quad \text{as } x \to S, \quad \text{for some } k > 0,$$

where $S = \partial B$ is the (full) topological boundary of B in R^n.

For $\alpha > 0$ and $0 < p < \infty$, we let $l^{p,\alpha-1}$ denote the closed subspace of $L^{p,\alpha-1} = L^p(B, d\nu_\alpha)$ consisting of harmonic functions in $L^{p,\alpha-1}$.

The purpose of the present paper is to study the Besov spaces B^p.

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