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A RELATION BETWEEN SMALL AMPLITUDE AND BIG LIMIT CYCLES

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ABSTRACT. There are two well-known methods for generating limit cycles for planar systems with a nondegenerate critical point of focus type: the degenerate Hopf bifurcation and the Poincaré-Melnikov method; that is, the study of small perturbations of Hamiltonian systems. The first one gives the so-called small amplitude limit cycles, while the second one gives limit cycles which tend to some concrete periodic orbits of the Hamiltonian system when the perturbation goes to zero (big limit cycles, for short). The goal of this paper is to relate both methods. In fact, in all the families of differential equations that we have studied, both methods generate the same number of limit cycles. The families studied include Liénard systems and systems with homogeneous nonlinearities.

1. Introduction and main results. One of the most interesting and difficult problems in the theory of planar differential equations is the control of the number of limit cycles that a differential equation or a family of differential equations can have. Two well-known methods used for generating limit cycles and hence for giving lower bounds for this number for a given family are: degenerate Hopf bifurcation and the Poincaré-Melnikov method; that is, the study of perturbations of Hamiltonian systems.

Although the above two methods are usually considered as independent, there have been several attempts to relate both for concrete families of differential equations. See the results of [**3**] on quadratic systems and the results of [**4**] on Liénard systems.

The main goal of this paper is to relate both approaches when we study the number of limit cycles surrounding a nondegenerate critical point. To be more precise, we need to introduce some notation.

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