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A DEGREE OF NONLOCAL CONNECTEDNESS

JANUSZ J. CHARATONIK AND WŁODZIMIERZ J. CHARATONIK

ABSTRACT. To any continuum X we assign an ordinal number (or the symbol ∞) $s(X)$, called the degree of nonlocal connectedness of X. We show that (1) the degree cannot be increased under continuous surjections; (2) for hereditarily unicoherent continua X, the degree of a subcontinuum of X is less than or equal to $s(X)$; (3) $s(C(X)) \leq s(X)$, where $C(X)$ denotes the hyperspace of subcontinua of a continuum X. We also investigate the degrees of Cartesian products and inverse limits. As an application we construct an uncountable family of metric continua X homeomorphic to $C(X)$.

Introduction. The idea of using ordinal numbers as a "measure" of some local or global properties of (compact) spaces is not new. Usually these properties are related to (non-)connectedness, and the defined "measure" can be used as a tool in studying various other properties of investigated spaces, both structural (internal) and mapping (external) ones. For example, Iliadis in [**14**] defines the notion of a normal sequence for hereditarily decomposable and hereditarily unicoherent metric continua (i.e., for λ-dendroids) as follows. Let X be such a continuum. A continuum $H \subset X$ is said to be in $\mathcal{I}(X)$ if, given any decomposition of X into finitely many subcontinua, H is contained in one element of the decomposition. Let $\Sigma = \{H_\alpha : \alpha < \lambda\}$ be a transfinite sequence of subcontinua of X, where λ is some countable ordinal number. Then Σ is called a *normal sequence* if (i) $H_0 = X$, (ii) $H_\beta = \mathcal{I}(H_\alpha)$ for ordinals $\beta = \alpha + 1$, (iii) $H_\beta = \cap\{H_\alpha : \alpha < \beta\}$ for limit ordinals β, and (iv) for each $\alpha < \lambda$ the continuum H_α is nondegenerate. The least upper bound (or minimum) $k(X)$ of the lengths of all normal sequences in X is called the *depth* of X. The concept was used to study various phenomena in λ-dendroids. For its modification, see [**31**].

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