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HANKEL TRANSFORMATION AND HANKEL CONVOLUTION OF TEMPERED BEURLING DISTRIBUTIONS

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ABSTRACT. In this paper we complete the distributional theory of Hankel transformation developed in [**5**] and [**18**]. New Fréchet function spaces $\mathcal{H}_\mu(w)$ are introduced. The functions in $\mathcal{H}_\mu(w)$ have a growth in infinity restricted by the Beurling type function w. We study on $\mathcal{H}_\mu(w)$ and its dual the Hankel transformation and the Hankel convolution.

1. Introduction. The Hankel integral transformation is usually defined by

$$h_\mu(\phi)(x) = \int_0^\infty (xy)^{-\mu} J_\mu(xy)\phi(y)y^{2\mu+1}\, dy, \quad x \in (0, \infty),$$

where J_μ represents the Bessel function of the first kind and order μ. We will assume throughout this paper that $\mu > -1/2$. Note that if ϕ is a Lebesgue measurable function on $(0, \infty)$ and

$$\int_0^\infty x^{2\mu+1}|\phi(x)|\, dx < \infty,$$

then, since the function $z^{-\mu}J_\mu(z)$ is bounded on $(0, \infty)$, the Hankel transform $h_\mu(\phi)$ is a bounded function on $(0, \infty)$. Moreover, $h_\mu(\phi)$ is continuous on $(0, \infty)$ and, according to the Riemann-Lebesgue theorem for Hankel transforms ([**17**]), $\lim_{x\to\infty} h_\mu(\phi)(x) = 0$.

The study of the Hankel transformation in distribution spaces was started by Zemanian ([**18**], [**19**]). In [**18**] the Hankel transform of distribution of slow growth was defined. More recently, Betancor and

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