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FAMILIES OF MAXIMAL SUBBUNDLES OF STABLE VECTOR BUNDLES ON CURVES

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ABSTRACT. Let X be a smooth projective curve of genus $g \geq 2$, and let E be a vector bundle on X. Let $M_k(E)$ be the scheme of all rank k subbundles of E with maximal degree. For every integer r, k and x with $0 < k < r$ and either $2k \leq r$ and $0 \leq x \leq (k-1)(r-2k+1)$ or $2k > r$ and $0 \leq x \leq (r-k-1)(2k-r+1)$, we construct a rank r stable vector bundle E such that $M_k(E)$ has an irreducible component of dimension x. Furthermore, if there exists a stable vector bundle F with small Lange's invariant $s_k(F)$ and with $M_k(F)$ 'spread enough,' then X is a multiple covering of a curve of genus bigger than 2.

1. Introduction. Let X be a smooth projective curve of genus $g \geq 2$ defined over an algebraically closed field $\mathbf{K}$. In this paper we study the rank r stable vector bundles, E, on X such that for some integer k with $0 < k < r$, E has a 'large' family of subbundles with rank k and maximal degree. For positive integers r, d let $M(X; r, d)$ be the moduli space of stable vector bundles on X of rank r and degree d. It is well known that $M(X; r, d)$ is smooth and irreducible. For a positive integer k with $0 < k < r$, let $M_k(E)$ be the set of all rank k subbundles of E with maximal degree. Being a Quot-scheme, $M_k(E)$ has a natural scheme-structure. For the intent of this paper we will only need to consider its reduced structure. Indeed, we are interested in finding a stable vector bundle E such that $M_k(E)$ has an irreducible component with prescribed dimension. Since every element in $M_k(E)$ has maximal degree, the scheme $M_k(E)$ is complete. Hence, by [**7**, pp. 254–255], we have $\dim(M_k(E)) \leq k(r-k)$ for every rank r vector bundle E. Fixing x with $x \leq k(r-k)$, it is very easy to find a decomposable rank r vector bundle E such that $M_k(E)$ has an irreducible component of dimension x. But we are interested in stable vector bundles which are indecomposable. Hence, using extensions of

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