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CUBIC FIELDS WITH A POWER BASIS

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ABSTRACT. It is shown that there exist infinitely many cubic fields L with a power basis such that the splitting field M of L contains a given quadratic field K.

1. Introduction. We prove the following result, which answers a question posed to the authors by James G. Huard.

Theorem. *Let K be a fixed quadratic field. Then there exist infinitely many cubic fields L with a power basis such that the splitting field M of L contains K.*

We remark that Dummit and Kisilevsky [**2**] have shown that there exist infinitely many cyclic cubic fields with a power basis.

2. Squarefree values of quadratic polynomials. The following result is due to Nagel [**5**]. We quote it in the form given by Huard [**3**].

Proposition 2.1. *Let $f(x)$ be a polynomial with integer coefficients such that*

(i) *the degree of $f(x) = k$,*

(ii) *the discriminant of $f(x)$ is not equal to zero,*

(iii) *$f(x)$ is primitive,*

(iv) *$f(x)$ has no fixed divisors which are kth powers of primes.*

Then infinitely many of $f(1), f(2), f(3), \ldots$ are kth power free.

We recall that a positive integer $d > 1$ is called a fixed divisor of the primitive polynomial $f(x) \in \mathbf{Z}[x]$ if $f(k) \equiv 0 \pmod d$ for all

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