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INDUCTION OF CHARACTERS, KERNELS AND LOCAL SUBGROUPS

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1. Introduction. Let G be a finite group, and let p be a prime number. Suppose that K is a subgroup of G having a character α such that $\alpha^G \in \mathrm{Irr}\,(G)$. If $\alpha(1) \leq p/2$, G.R. Robinson proved in [**5**] that

$$(\alpha_{\mathbf{N}_K(P)})^{\mathbf{N}_G(P)} \in \mathrm{Irr}\,(\mathbf{N}_G(P)),$$

where $P \in \mathrm{Syl}_p(K)$.

In [**2**], M. Isaacs proved this result when $\alpha(1) = 1$ by using elementary character theory. Robinson's general proof uses Green theory.

In the present note, we take a different approach and pay attention to the group $K/\ker(\alpha)$ instead of the degree of α.

Theorem A. *Let $K \subseteq G$ and suppose that $\alpha \in \mathrm{Irr}\,(K)$ induces $\alpha^G \in \mathrm{Irr}\,(G)$. Suppose that $P \in \mathrm{Syl}_p(K)$ is such that $P\ker(\alpha) \triangleleft K$. Then*

$$(\alpha_{\mathbf{N}_K(P)})^{\mathbf{N}_G(P)} \in \mathrm{Irr}\,(\mathbf{N}_G(P)).$$

Notice that if α is linear then $K/\ker(\alpha)$ is abelian, and we are in the hypothesis of Theorem A. Also, by using the Feit-Thompson theorem on linear groups, we will recover most of Robinson's theorem. The only case in the Robinson's situation which is not treated by our methods is when $\alpha(1) = (p-1)/2$. In this case, in view of the classification of finite simple groups, a description of the non-p-closed linear groups of degree $(p-1)/2$ is possible. As remarked by the referee, it might be possible to weaken the hypothesis of p-closure in Theorem A to still obtain the same conclusion.

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