ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 3, Fall 2001

WITT SUBGROUPS AND CYCLIC DIEUDONNÉ MODULES KILLED BY p

ALAN KOCH

ABSTRACT. Let k be a perfect field of characteristic $p > 0$. We obtain a complete classification of cyclic Dieudonné modules killed by p, which in turn gives us a complete classification of Witt subgroups killed by p. Finally, we construct an explicit formula for the number of Witt subgroups of a given dimension over a finite field k and use this to disprove a conjecture of Lubin.

Let k be a perfect field of characteristic $p > 0$. Let G be an affine finite commutative k-group scheme. It is well known (e.g., [**11**]) that G splits into the direct sum of G' and G'', where G' is the maximal connected unipotent subgroup of G. While G'' can be classified using descent (over an algebraically closed field it decomposes into various roots of unity functors and constant group schemes of dimension p^r for various r), the classification of connected unipotent group schemes is more difficult.

To fully understand this subcategory of group schemes, we must introduce Dieudonné modules. Dieudonné modules are a class of modules (over a certain ring E) which correspond to these group schemes. It is for this reason that we are interested in studying Dieudonné modules. In this paper we will use Dieudonné modules to obtain a complete classification of Witt subgroups that are killed by p. A Witt subgroup refers to a finite subgroup scheme of any finite-length Witt vector group scheme. In the first section we describe the basic correspondence between group schemes and Dieudonné modules; we give additional properties for the Dieudonné modules that correspond to Witt subgroups killed by p, namely, the modules that are killed by p and are cyclic. In Sections 2 and 3 we classify all of the modules that correspond to Witt subgroups; we see that they can be parameterized by triples (l, m, η), where l and m are positive integers and η is an

1991 AMS *Mathematics Subject Classification.* 14L20.
Received by the editors on August 9, 1999, and in revised form on September 1, 2000.