ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 3, Fall 2001

IRREDUCIBLE CONTINUA OF TYPE λ WITH ALMOST UNIQUE HYPERSPACE

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ABSTRACT. For an irreducible continuum X of type λ we study the family of all continua Y for which hyperspaces of subcontinua $C(X)$ and $C(Y)$ are homeomorphic. The family is determined if each layer of X is a layer of cohesion and the set of degenerate layers is dense in X.

1. Introduction. Given a (metric) continuum X, denote by $C(X)$ the hyperspace of subcontinua of X (i.e., the family of all subcontinua of X) metrized by the Hausdorff metric. A class Λ of continua is said to be *C-determined* (see [**16**, p. 33]), provided that for every $X, Y \in \Lambda$ if the hyperspaces $C(X)$ and $C(Y)$ are homeomorphic, then so are the continua X and Y. For various results on this subject, see e.g., [**16**, pp. 32–33], [**10**, pp. 437–438], [**8**], [**9**], [**14**] and [**15**]. The following concept is closely related to the above.

For a given continuum X, consider a family $\Im(X)$ of continua Y such that:

(1.1) no two distinct members of $\Im(X)$ are homeomorphic,

(1.2) $C(Y)$ is homeomorphic to $C(X)$ for each member Y of $\Im(X)$,

(1.3) $\Im(X)$ is the maximal family satisfying conditions (1.1) and (1.2), i.e., if Z is a continuum such that $C(Z)$ is homeomorphic to $C(X)$, then Z is homeomorphic to Y for some $Y \in \Im(X)$.

A continuum X is said to have *unique hyperspace* provided that the family $\Im(X)$ consists of one element only, viz. of X, [**1**, Definition 1]; *almost unique hyperspace* provided that the family $\Im(X)$ is finite and consists of more than one element, [**2**, Definition 1.1].

2000 *Mathematics Subject Classification.* 54B20, 54F15, 54F50.
Key words and phrases. Arc-like, compactification, continuum, homeomorphism, hyperspace, irreducible, type λ, ray, unique hyperspace.
Received by the editors on April 14, 2000.