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ON THE WEAK PROPERTY OF LEBESGUE OF $L^1(\Omega, \Sigma, \mu)$

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ABSTRACT. A Banach space X is said to have the *weak property of Lebesgue* if every Riemann integrable mapping from the closed interval $[a, b]$ to X is weakly continuous almost everywhere on $[a, b]$. In this paper we prove that if (Ω, Σ, μ) is a totally finite, complete and countably generated measure space, then $L^1(\Omega, \Sigma, \mu)$ has the weak property of Lebesgue.

In this paper we are concerned with the Riemann integration in Banach spaces which was first studied by Graves [**2**]. In [**3**], Gordon compiled many results of Graves and others, e.g., Alexiewicz and Orlicz [**1**] and studied the interesting problem of determining which Banach spaces X have the property of Lebesgue, that is, the property that every Riemann integrable mapping from $[a, b]$ to the space X is continuous almost everywhere. After encouragement in 1992 by Professor Joe Diestel of Kent State University, we studied some problems related to the Riemann integration in Banach spaces and in [**6**] established some new characterizations of the Schur property and the H property of Banach spaces using Riemann integration. Inspired mainly by [**3**], in [**5**] we introduced the concept of the weak property of Lebesgue of a Banach space X and did some preliminary study, in which we pointed out that the most familiar Banach spaces enjoy the weak property of Lebesgue.

In this paper we assume that (Ω, Σ, μ) is a totally finite, complete and countably generated measure space. Since (Ω, Σ, μ) is countably generated, there is a sequence $\{G_k\} \subset \Sigma$ such that for any $E \in \Sigma$, $\delta > 0$, there exists some G_k satisfying $\mu(E \Delta G_k) < \delta$, see [**4**, pp. 168–169]. Under the stated assumptions we prove that $L^1(\Omega, \Sigma, \mu)$ has the weak property of Lebesgue. This result, of course, implies that the well-known space $L^1[0, 1]$ has the weak property of Lebesgue.

Key words and phrases. Riemann integration, Banach space, weak property of Lebesgue, $L^1(\Omega, \Sigma, \mu)$.
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