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SEQUENCE SPACES OF CONTINUOUS FUNCTIONS

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1. Introduction. The familiar sequence spaces c_0 and l^p, $0 < p < \infty$, may conveniently be understood as spaces of continuous functions that vanish at infinity on the locally compact space $\mathbf{N}$ of positive integers, or as spaces of continuous functions on the one-point compactification of $\mathbf{N}$. On the other hand, to study l^∞ as a space of continuous functions on a compact space (rather than continuous bounded functions on $\mathbf{N}$) requires considerably more effort; the appropriate compact space is $\beta\mathbf{N}$, the Stone-Čech compactification of $\mathbf{N}$, and it may be obtained variously by methods of set theory, topology, or analysis.

If instead of looking at sequences of scalars we look at sequences whose entries are taken from some fixed Banach space E, the study of c_0 and l^p, $0 < p < \infty$, becomes only marginally more involved. There is a compact space X (for instance, the closed unit ball of the dual space of E, in the weak* topology) such that E may be regarded as a space of continuous functions on X, and members of the sequence spaces can be interpreted as continuous functions on the one-point compactification of $\mathbf{N} \times X$. However, viewing l^∞ as a space of continuous functions now often involves significant new complexities which can have important reverberations elsewhere in analysis. It is this situation that we propose to explore here.

By a *Banach function space* on a compact Hausdorff space X, we mean a Banach space lying in $C(X)$ which separates the points of X, has norm dominating the uniform norm, and (for convenience) contains the constant functions. Here point separation means that if x_1 and x_2 are distinct points of X, then $f(x_1) \neq f(x_2)$ for some function f in the function space, and norm domination means that there is a positive constant c such that the function space norm on f is at least $c\|f\|_\infty$, where $\|f\|_\infty = \sup\{|f(x)| : x \in X\}$. Scalars may be real or complex.

If E is a Banach function space on X, we denote by $l^\infty(\mathbf{N}, E)$, or more

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