ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 2, Summer 2001

FINITE PROJECTIONS IN MULTIPLIER ALGEBRAS

E. PARDO

A la Fina.

ABSTRACT. We give a characterization of finiteness of projections in the multiplier algebra of a σ-unital C^*-algebra of real rank zero and stable rank one.

1. Introduction. The behavior and properties of projections are some of the most interesting topics in the theory of C^*-algebras and also objects of intensive study (see [**3**] for a complete survey on this topic).

This is particularly important in the case of C^*-algebras with real rank zero, a class introduced by Brown and Pedersen in 1991, [**5**], although this property, under different names, was the object of intensive study some years ago (e.g., [**4**] or [**14**]). This class includes AF C^*-algebras, von Neumann algebras, Rickart C^*-algebras, irrational rotation algebras and purely infinite simple C^*-algebras among others (see, for example, [**5**], [**6**], [**20**]), and because of [**5**, Theorem 2.6] the structure of these algebras is closely related to the structure and properties of their projections.

One of the points of interest on this topic is to know whether projections in a C^*-algebra A are finite or not. Recall that a projection $p \in A$ is *infinite* if there exist p', q' nonzero orthogonal subprojections of p such that $p' + q' = p$ and $p' \sim p$ (where "$\sim$" means Murray-von Neumann equivalent), and otherwise we say that p is *finite*. Also, if there exists a projection $q \in A$ such that $2 \cdot p \oplus q \sim p$ (where "$\oplus$" means orthogonal sum, viewing the projection in $M_\infty(A)$, see [**2**, Chapter 5]), then we say that p is *properly infinite*. The existence

1991 AMS *Mathematics Subject Classification.* 46L05, 19K14.
Key words and phrases. Real rank zero, finite projection, Riesz monoid.
Partially supported by DGESIC grant no. PB98-0873, by the Comissionat per Universitats i Recerca de la Generalitat de Catalunya, by the Plan Andaluz de Investigación and by the Plan Propio de la Universidad de Cádiz.
Received by the editors on June 15, 1999, and in revised form on February 7, 2000.