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SIMPLE GEOMETRIC CHARACTERIZATION OF SUPERSOLVABLE ARRANGEMENTS

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1. Introduction. An arrangement of hyperplanes is a finite collection of $\mathbf{C}$-linear subspaces of dimension $(l-1)$ in $\mathbf{C}^l$. For such an arrangement $\mathcal{A}$, there is a natural projective arrangement $\mathcal{A}^*$ of hyperplanes in $\mathbf{CP}^{l-1}$ associated to it. Let $M(\mathcal{A}) = \mathbf{C}^l - \cup\{H : H \in \mathcal{A}\}$ and $M(\mathcal{A}^*) = \mathbf{CP}^{l-1} - \cup\{H^* : H^* \in \mathcal{A}^*\}$. Then it is clear that $M(\mathcal{A}) = M(\mathcal{A}^*) \times \mathbf{C}^*$. The central problem in the theory of arrangements is to find a connection between the topology or differentiable structure of $M(\mathcal{A})$, respectively $M(\mathcal{A}^*)$, and the combinatorial geometry of $\mathcal{A}$, respectively $\mathcal{A}^*$.

More specifically, we would like to know the homotopy properties of $M(\mathcal{A})$ and how these properties relate to various other well-known properties of arrangements. Many people have asked the following questions. Precisely when is $M(\mathcal{A})$ a $K(\pi,1)$ space?

In [**2**], Brieskorn considers the Coxeter group W acting on $\mathbf{R}^l$. W also acts as a reflection group in $\mathbf{C}^l$. Let $\mathcal{A} = \mathcal{A}(W)$ be its reflection arrangement. Brieskorn conjectured that $\mathcal{A}(W)$ is a $K(\pi,1)$ arrangement for all Coxeter groups W. He proved this for some of the groups by representing M as the total space of a sequence of fibrations. Deligne [**3**] settled the question by proving that the complement of complexification of a real simplicial arrangement is $K(\pi,1)$. This result proves Brieskorn's conjecture because the arrangement of a Coxeter group is simplicial. Recently, Jambu and Terao [**4**] introduced the property of supersolvability of an arrangement. This property is combinatorial in nature, that is, it depends only on the pattern of intersection of the hyperplanes or equivalently on the lattice associated to the arrangement. It turns out that complement $M(\mathcal{A})$ of a supersolvable arrangement is the total space of a fiber bundle in which the base and fiber are $K(\pi,1)$ spaces. The long exact homotopy sequence of the bundle shows that

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