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PARAQUATERNIONIC KÄHLER MANIFOLDS

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ABSTRACT. Paraquaternionic Kähler manifolds are studied with special attention to their curvatures. Since such manifolds are necessarily pseudo-Riemannian of neutral signature, the behavior of the Jacobi operators along spacelike, timelike and null geodesics is considered separately, for the purpose of study on Osserman problems on paraquaternionic Kähler manifolds.

1. Introduction. The study of Jacobi operators, as a basic part of the curvature tensor, is one of the central topics in Riemannian and pseudo-Riemannian geometry. Since Jacobi operators provide a way to measure the geodesic deviation, it is natural to expect that their properties strongly influence the geometry of manifolds. Osserman conjectured in his study [**30**] of those Riemannian manifolds, whose Jacobi operators have constant eigenvalues on the unit sphere bundle, that they must be locally flat or rank-one symmetric. This was shown by Chi in many cases, although the general problem still remains open. (See [**9**], [**10**], [**11**] and [**21**] for more details and further references). In the pseudo-Riemannian setting, the Osserman problem was firstly studied in the framework of Lorentzian geometry, showing that it is equivalent to constant curvature [**4**], [**17**], [**18**]. The situation is however much more complicated for pseudo-Riemannian metrics of other signatures. Indeed, in the pseudo-Riemannian setting, non-symmetric Osserman manifolds of signature (p, q), $p, q > 1$, exist. See [**19**]. In [**5**] a systematic study of the Osserman problem for metrics of signature $(++--)$ is developed. If the Jacobi operators are assumed to be diagonalizable, such spaces correspond to the indefinite real and

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