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ORBIFOLD SPECTRAL THEORY

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ABSTRACT. In this paper we study Sobolev spaces for smooth closed orientable Riemannian orbifolds. In particular we prove the Sobolev embedding theorem, the Rellich-Kondrakov theorem and Poincare's inequalities. From these theorems we derive properties of the spectrum of the Laplacian. In particular, Weil's asymptotic formula and estimates from below of the eigenvalues of the Laplacian are proved in analogy with the manifold case.

0. Introduction. In this paper we study spectral theory for closed orientable orbifolds. (In the literature orbifolds are also called V-manifolds.) Orbifold Hilbert Sobolev spaces H_k^2 were first introduced by Chiang in [**5**]. Other orbifold Sobolev spaces are also considered in [**12**]. After defining general Sobolev spaces for closed orientable orbifolds, we establish Sobolev embedding theorems and the Rellich-Kondrakov theorem. By using these theorems we prove, in analogy with the manifold case, Weil's asymptotic formula for the eigenvalues of the orbifold Laplacian. We also prove Poincare's inequalities. Our presentation of this material follows closely [**9**], [**1**], which deal with the manifold case. By proving more refined Sobolev inequalities we also obtain estimates from below of the eigenvalues of the Laplacian, generalizing the results of [**4**] to the orbifold case.

This paper is the starting point of an ongoing project aiming at generalizing several well-known results of spectral theory for manifolds to orbifolds.

We will now recall a few basic definitions used throughout the paper [**10**], [**5**], [**7**]. Unless otherwise specified, all our orbifolds are assumed to be both smooth and Riemannian.

A closed orientable orbifold, M, can be covered by a finite number of charts $(\Omega_l, \phi_l)_{l=1,\dots,N}$, where $\Omega_l = \tilde{\Omega}_l/G_l$ with $\tilde{\Omega}_l$ homeomorphic to $\mathbf{R}^n$ and G_l a finite subgroup of $SO(n)$. The local lifts of the changes of charts are assumed to be smooth.

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