ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 1, Spring 2001

GEOMETRICAL PROPERTIES OF THE PRODUCT OF A C^*-ALGEBRA

KAIDI EL AMIN, ANTONIO MORALES CAMPOY AND
ANGEL RODRIGUEZ PALACIOS

0. Introduction. The study of the geometry of norm-unital complex Banach algebras at their units [**5**], [**6**] takes its first impetus from the celebrated Bohnenblust-Karlin theorem [**3**] asserting that the unit of such an algebra A is a vertex of the closed unit ball of A. As observed in [**5**, pp. 33–34], the Bohnenblust-Karlin paper contains a stronger result, namely that, for such an algebra A, the inequality $n(A, \mathbf{1}) \geq (1/e)$ holds. Here $\mathbf{1}$ denotes the unit of A, and $n(A, \mathbf{1})$ is a suitably defined nonnegative real number which depends only on the Banach space of A and the norm-one distinguished element $\mathbf{1}$. As the main result, we prove in this paper that the product of every nonzero C^*-algebra A is a vertex of the closed unit ball of the Banach space $\Pi(A)$ of all continuous bilinear mappings from $A \times A$ into A. As in the above mentioned case, the vertex property follows from stronger "numerical" conditions. Indeed, if A is a nonzero C^*-algebra, and if p_A denotes the product of A, then $n(\Pi(A), p_A)$ is equal to 1 or 1/2 depending on whether or not A is commutative (Theorem 1.1). We note that our main result improves the recent one in [**24**, Corollary 2.7] asserting that the product of every nonzero C^*-algebra A is an extreme point of the closed unit ball of $\Pi(A)$.

In Section 2 we show that the main result remains true for the so-called alternative C^*-algebras (Theorem 2.5). Alternative C^*-algebras are defined by means of the Gelfand-Naimark abstract system of axioms but relaxing the familiar requirement of associativity to that of alternativity. Alternative C^*-algebras arise in a natural way in functional analysis. Indeed, Gelfand-Naimark axioms on a general nonassociative unital algebra imply the alternativity [**22**, Theorem 14] (see also [**9**]) and the existence of alternative C^*-algebras failing to be associative is well known (see [**17**, Example 13] and [**8**, Theorem 3.7]). Alternative C^*-algebras are studied in detail in [**20**] and [**8**] and have shown

Received by the editors on July 17, 1999.