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COMPOSITION OPERATORS FROM THE SPACE OF CAUCHY TRANSFORMS INTO ITS HARDY-TYPE SUBSPACES

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ABSTRACT. This paper studies the boundedness and compactness of composition operators from the space of Cauchy transforms into its Hardy-type subspaces. They are characterized by the behavior of the Cauchy kernel composed by inducing self-maps of the unit disk.

1. Introduction. Let $\mathbf{D}$ be the open unit disk and T the unit circle in the complex plane. A holomorphic function f on $\mathbf{D}$ is said to belong to K, the space of all Cauchy transforms, if it admits a representation $f(z) = \int_T 1/(1-\bar{\eta}z)\, d\mu(\eta)$ where μ is a complex Borel measure on T. The following inclusion relations between the class K and Hardy spaces are well known: $H^1 \subset K \subset \cap_{p<1} H^p$. See [**2**] and [**9**].

Now let φ be a holomorphic self-map of $\mathbf{D}$. It was known that the composition operator $C_\varphi(f) = f \circ \varphi$ acts as a bounded operator on the Hardy spaces [**12**], [**10**] and on the space K [**2**]. The compactness of C_φ on the Hardy spaces was completely characterized in terms of the behavior of Nevanlinna counting function by Shapiro [**14**]. A few years later another equivalent characterization, the so-called Sarason's condition, was obtained by Sarason, Shapiro and Sundberg [**13**], [**15**]. Recently, Cima and Matheson [**4**] considered the problem of characterizing the compactness of C_φ on K and have established that C_φ is compact on K if and only if it is compact on H^2.

The purpose of this paper is to study composition operators C_φ which map the space K into some of its subspaces. Indeed, we shall characterize those holomorphic self-maps φ of $\mathbf{D}$ that induce bounded or compact composition operators from the space K to H^p, $p \geq 1$,

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