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HOMOGENEOUS ALGEBRAIC DISTRIBUTIONS

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ABSTRACT. Vertical distributions on a vector bundle admitting a system of homogeneous algebraic vector fields of the same degree are characterized.

1. Introduction. The goal of this paper is to provide a characterization of homogeneous algebraic distributions on vector bundles. The starting point is the classical result according to which a vector field X on $\mathbf{R}^m$ is homogeneous algebraic of degree d if and only if $[\chi, X] = (d-1)X$, χ being the Liouville vector field. If one wants to involve exclusively the module structure spanned by a vector field X in characterizing algebraic vector fields, then one is led to study the equation $[\chi, X] = fX$. In this case a first result (cf. 4.7 below) states that the function f should be constant along the zero section of the vector bundle $p : E \to M$ on which X is defined, and this constant should be an integer ≥ -1. Our main result is a generalization of the above statement to distributions of arbitrary rank: it is stated that a vertical distribution locally spanned by $X_1, \ldots, X_r$ is homogeneous algebraic of degree d if and only if an $r \times r$ matrix $A = (a_{ij})$, $a_{ij} \in C^\infty(E)$, exists which is equal to $d-1$ times the identity matrix along the zero section of E, and such that $[\chi, X_j] = \sum_{i=1}^r a_{ij} X_i$, for $j = 1, \ldots, r$ (cf. 4.6).

Algebraic distributions play a role in several fields of real and complex geometry such as singularities of vector fields, the moduli problem for differential forms, calculation of differential invariants of a Lie group action, etc. (e.g., see [**6**], [**10**], [**14**], [**16**], [**22**]). Thus, it seems interesting to obtain a characterization of these differential systems. Linear representations of families of Lie groups on vector bundles give rise to such distributions in a natural way. Families of Lie groups and specially Lie group fiber bundles naturally appear in the field theory

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