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SQUARES OF RIESZ SPACES

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ABSTRACT. In this paper we provide three approaches to the notion of squares of Riesz spaces and show that they are equivalent.

The study of powers of Banach lattices was initiated by Lozanovsky in [**8**] and a similar construction was studied by Krivine in [**5**], of which an account can be found in [**6**] as well. In a recent paper [**9**], partially rooted in probability theory, Szulga introduces the notion of powers of uniformly complete Riesz spaces. Avoiding a technical description, which for all of the above authors involves functional calculus, their results are exemplified by the fact that (in Szulga's notation)

$$(L^1)^2 = L^2.$$

For reasons that will become clear, we are interested in Szulga's power 1/2 rather than his power 2 and, confusing as it may seem at first, Szulga's power 1/2 will be called power 2 by us and consequently our theory will be exemplified by (in our notation)

$$(L^1)^2 = L^{1/2}.$$

Our theory develops squares of any Archimedean vector lattices. These squares of vector lattices play a role in a surprising variety of theories in functional analysis. We were motivated to investigate them while studying certain Riesz algebras and orthosymmetric operators in [**1**] and [**2**]. Given a uniformly complete f-algebra E, we defined its square in [**2**] to be

$$E^2 = \{fg : f, g \in E\},$$

which may explain the notation that we prefer. It is known that Riesz spaces can be embedded in (semi-prime) f-algebras of the type

$$C^\infty(X).$$

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