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EXISTENCE OF THREE SOLUTIONS TO INTEGRAL AND DISCRETE EQUATIONS VIA THE LEGGETT WILLIAMS FIXED POINT THEOREM

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ABSTRACT. Criteria are developed for the existence of three nonnegative solutions to integral and discrete equations. The strategy involves using the Leggett Williams fixed point theorem.

1. Introduction. In this paper we present results which guarantee the existence of three nonnegative solutions to integral and discrete equations. The results we establish are new since this is the first paper, to our knowledge, that discusses the existence of three nonnegative solutions to integral equations. In addition, the results in this paper contain almost all results in the recent papers [**3**–**6, 8, 9**] on the existence of three solutions to higher order differential and difference equations since we make full use of the properties of the concave functional on the cone. Indeed, if we assume the conditions in [**3**–**6, 8, 9**], then the conditions in this paper are trivially satisfied.

For the remainder of the introduction we present some preliminaries which will be needed in Sections 2 and 3. Let $E = (E, \|.\|)$ be a Banach space and $C \subset E$ a cone. By a concave nonnegative continuous functional ψ on C we mean a continuous mapping $\psi : C \to [0, \infty)$ with

$$\psi(\lambda x + (1-\lambda)y) \geq \lambda\psi(x) + (1-\lambda)\psi(y)$$
$$\text{for all } x, y \in C \quad \text{and } \lambda \in [0,1].$$

Let $K, L, r > 0$ be constants with C and ψ as defined above. We let

$$C_K = \{y \in C : \|y\| < K\}$$

and

$$C(\psi, r, L) = \{y \in C : \psi(y) \geq r \text{ and } \|y\| \leq L\}.$$

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