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ON A CLASS OF HILBERT-SCHMIDT OPERATORS

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ABSTRACT. We consider a class of integral operators with L^2 kernel which are often encountered in the resolution of boundary value problems and in scattering theory. We prove that, under certain conditions, these operators are contractions; applications of this result to a nonlinear differential equation at resonance and to an integral equation of inverse scattering theory are discussed.

0. Introduction. In the applications of integral equations methods to differential equations and scattering theory, one often deals with integral operators K with L^2 kernel $\mathcal{K}$ of the following type:

$$\mathcal{K}(x,y) = U(x)\,\Gamma(x,y)V(y), \quad x,\, y \in \Omega \subseteq \mathbf{R}^n, \tag{1.1}$$

where $U,\ V \in L^\infty(\Omega)$ and the function $\Gamma(x,y)$ is the kernel of a bounded integral operator (not necessarily compact). In this note, we discuss sufficient conditions on the functions Γ, U and V for K being a contraction.

Although the result follows easily by applying well known arguments of Functional Analysis, it may be useful for the solution of certain integral equations with L^2 kernels, related to boundary value problems and scattering theory. We will discuss two applications: in the first one, we provide a non-resonance condition for a nonlinear second order differential equation, which guarantees *unique solvability*; in the second example, we derive an alternative proof of the resolubility of the *Marchenko integral equation* [**6**], which allows to solve the inverse scattering problem for the Schrodinger equation.

1. Statement and proof of the main result. We prove our result in the following form:

Theorem 1.1. *Let $K\ :\ L^2(\Omega) \to L^2(\Omega)$ be a Hilbert-Schmidt operator with kernel $\mathcal{K}(x,y) = U(x)\,\Gamma(x,y)V(y)$. Assume that*: i) U, V *are functions in $L^\infty(\Omega)$, with $||U||_\infty \le 1$, $||V||_\infty \le 1$.*