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FINITE ELEMENT APPROXIMATION WITH QUADRATURE TO A TIME DEPENDENT PARABOLIC INTEGRO-DIFFERENTIAL EQUATION WITH NONSMOOTH INITIAL DATA

AMIYA K. PANI AND RAJEN K. SINHA

ABSTRACT. In this paper we analyze the effect of numerical quadrature in the finite element analysis for a time dependent parabolic integro-differential equation with nonsmooth initial data. Both semi-discrete and fully discrete schemes are discussed and optimal order error estimates are derived in $L^\infty(L^2)$ and $L^\infty(H^1)$ norms using energy method when the initial function is only in H_0^1. Further, quasi-optimal maximum norm estimate is shown to hold for rough initial data.

1. Introduction. In this paper we consider a finite element Galerkin method with spatial quadrature for the following time dependent parabaolic integro-differential equation

$$
\begin{aligned}
u_t + A(t)u &= \int_0^t B(t,s)u(s)\,ds \quad \text{in } \Omega \times J,\\
u &= 0 \quad \text{on } \partial\Omega \times J, \qquad (1.1)\\
u(\cdot,0) &= u_0 \quad \text{in } \Omega,
\end{aligned}
$$

where Ω is a convex bounded domain in R^2 with boundary $\partial\Omega$, $u(x,t)$ is a real valued function in R^2 and J denotes the time interval $(0,T]$ with $T < \infty$. Here $A(t)$ is a time dependent, positive definite, self-adjoint and uniformly elliptic second order partial differential operator and $B(t,s)$ is a general second order partial differential operator, both with smooth coefficients.

Let $H_0^1(\Omega) = \{\phi \in H^1(\Omega) : \phi = 0 \text{ on } \partial\Omega\}$, and let $A(t;\cdot,\cdot)$ and $B(t,s;\cdot,\cdot)$ be the bilinear forms associated with the operators A and B, respectively. The weak formulation of the problem (1.1) is defined

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