JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 13, Number 1, Spring 2001

THE EXTRAPOLATION METHOD FOR TWO-DIMENSIONAL VOLTERRA INTEGRAL EQUATIONS BASED ON THE ASYMPTOTIC EXPANSION OF ITERATED GALERKIN SOLUTIONS

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ABSTRACT. In this paper we study the numerical solution of two-dimensional Volterra integral equations by Galerkin and the iterated Galerkin method. Asymptotic error expansion of the iterated Galerkin solution is obtained. We show that when piecewise polynomials of $\pi_{p-1,q-1}$ are used, the iterated Galerkin solution admits an error expansion in powers of the stepsizes h and k, beginning with terms in h^{2p} and k^{2q}. Thus, Richardson's extrapolation can be performed based on this error expansion, and this will increase the accuracy of the numerical solution greatly. The theoretical results are confirmed by some numerical experiments.

1. Introduction. In this paper we are concerned with the Galerkin method and the iterated Galerkin method for the two-dimensional Volterra integral equation of the second kind

$$u(x,y) = g(x,y) + \int_0^x \int_0^y K(x,y,t,s)u(t,s)\,dt\,ds, \quad (x,y) \in D, \tag{1.1}$$

where $g(x,y)$, $K(x,y,t,s)$ are given continuous functions defined, respectively, on $D = [0,X] \times [0,Y]$ and $E = \{(x,y,t,s) : 0 \le t \le x \le X,\ 0 \le s \le y \le Y\}$. It follows from the classical theory of Volterra (see, for example, [**2**], [**3**]) that (1.1) possesses a unique solution $u^*(x,y) \in C(D)$. Especially when g and K are r times continuously differentiable on D and E, respectively, then u^* is r times continuously differentiable on D.

1991 AMS *Mathematics Subject Classification.* 65R20.
Key words and phrases. Volterra integral equations, Galerkin method, asymptotic expansion, Richardson extrapolation.
The work of the first author was supported in part by the National Natural Science Foundation of China, by the Natural Science Foundation of Guangdong Province and by the Scientific Research Foundation for Returned Overseas Chinese Scholars, State Education Ministry.
Received by the editors on January 4, 2000, and in revised form on November 1, 2000.