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# A LOWER ESTIMATE FOR THE NORM OF THE KERZMAN-STEIN OPERATOR

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ABSTRACT. We establish an elementary lower estimate for the norm of the Kerzman-Stein operator for a smooth, bounded domain. The estimate involves the boundary length and logarithmic capacity. The estimate is tested on model domains for which the norm is known explicitly. It is shown that the estimate is sharp for an annulus and a strip, and is asymptotically sharp for an ellipse and a wedge.

**1. Introduction.** Suppose $\Omega \subset\subset \mathbf{C}$ is a continuously differentiable, multiply connected domain in the plane and $L^2(\partial\Omega)$ is the space of square-integrable functions defined with respect to arclength measure on the boundary. The Cauchy singular operator on $L^2(\partial\Omega)$ can be expressed using a principal value integral,

$$\mathcal{C}_0 f(z) = \frac{1}{2\pi i}\,\text{P.V.}\int_{\partial\Omega} \frac{f(w)\,dw}{w-z} \quad \text{for} \quad z \in \partial\Omega.$$

It is known classically that $\mathcal{C}_0$ is bounded on $L^2(\partial\Omega)$, so its skew-hermitian part, $\mathcal{A} = \mathcal{C}_0 - \mathcal{C}_0^*$, is also bounded. In fact, Lanzani [**8**] showed that with these conditions $\mathcal{A}$ is compact–it acts by integration against the kernel,

$$A(z,w) = \frac{1}{2\pi i}\left[\frac{T(w)}{w-z} - \frac{\overline{T(z)}}{\overline{w}-\overline{z}}\right] \text{ for } w, z \in \partial\Omega,$$

where $T(w)$ is the positively-oriented unit tangent vector at $w \in \partial\Omega$. (The apparent singularities cancel each other.) The operator $\mathcal{A}$ is known as the Kerzman-Stein operator for $\Omega$. Kerzman and Stein used this operator to give an explicit construction for the Szegő kernel and, in so doing, they found an elegant way to compute the Riemann map [**7**].

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