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A NOTE ON NON-AUTONOMOUS IMPLICIT INTEGRAL EQUATIONS WITH DISCONTINUOUS RIGHT-HAND SIDE

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ABSTRACT. Let $I := [0,1]$, $f : I \times [0,\sigma] \to \mathbf{R}$, $g : I \times I \to [0,+\infty[$ and $h : I \times]0,+\infty[\to \mathbf{R}$. In this note we prove an existence result for solutions $u \in L^s(I)$ of the integral equation

$$h(t,u(t)) = f\left(t\,, \int_I g(t,z)\,u(z)\,dz \right) \text{ for a.a. } t \in I$$

where, in particular, the continuity of f with respect to the second variable is not assumed. Our result is a partial extension of a previous result of the same authors [**1**], where the function h was not allowed to depend explicitly on t.

1. Introduction. Let $A \subseteq]0,+\infty[$, $I := [0,1]$ and $J := [0,\lambda]$, with $\lambda > 0$, and let us consider the integral equation

$$h(u(t)) = f\left(t, \int_I g(t,z)\,u(z)\,dz \right) \quad \text{for a.a. } t \in I, \tag{1}$$

where $f : I \times J \to \mathbf{R}$, $g : I \times I \to [0,+\infty[$ and $h : A \to \mathbf{R}$. Recently, in the papers [**1, 2, 3, 6**], the equation (1) has been investigated together with some of its special cases, obtaining some existence results where the function f is not assumed to be continuous in the second variable. More specifically, in the paper [**1**] the function f was assumed to satisfy the following assumption: there exist a function $f^* : I \times J \to \mathbf{R}$ and two negligible sets $E_1, E_2 \subseteq J$, with E_2 closed, such that $f^*(\,\cdot\,,x)$ is measurable for each x in a countable dense subset of J and, for a.a. $t \in I$, one has

$$\left\{ x \in J : \ f^*(t,x) \neq f(t,x) \right\} \subseteq E_1,$$
$$\left\{ x \in J : \ f^*(t,\cdot\,) \text{ is discontinuous at } x \right\} \subseteq E_2.$$

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