

AN INTEGRAL EQUATION FOR
MAXWELL'S EQUATIONS IN A LAYERED
MEDIUM WITH AN APPLICATION
TO THE FACTORIZATION METHOD

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ABSTRACT. In the first part of this paper we study the direct scattering problem for time harmonic electromagnetic fields in a layered medium where arbitrary incident fields are scattered by a medium described by a space dependent permittivity and conductivity. We derive an integral equation and prove a theorem of Riesz-Fredholm type. In the second part we investigate the Factorization Method for the corresponding inverse problem with magnetic dipoles as incident fields. This is the problem to recover the support of the contrast from field measurements.

1. Introduction. This paper studies the direct and the inverse scattering problem for electromagnetic time harmonic fields where the scattering medium is described by a space-dependent permittivity and conductivity imbedded in a two-layered space. Following the standard notations we combine these quantities by introducing a space-dependent and complex valued relative permittivity ε_r . This scattering problem is motivated by a project¹ supported by the German Federal Ministry of Education and Research. Here, the two layers correspond to air (upper layer) and soil (lower layer), and the mine is modeled by a different permittivity and/or conductivity.

For a mathematical treatment of the direct problem we first refer to the monographs of Monk [20] and Nédélec [21] who proposed variational formulations with nonlocal boundary conditions on an artificial boundary based on the representation formula of Stratton-Chu type, cf. [6], or the exterior “capacity operator,” respectively. We refer also to [7] and [16] for earlier results of this kind. Although these approaches are presented for perfect conductors it is quite obvious how to carry them over to the case of an inhomogeneous medium which we characterize by the change of the electric permittivity $\varepsilon\varepsilon_r$ with respect to the

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