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A HYBRID METHOD FOR INVERSE BOUNDARY VALUE PROBLEMS FOR AN INCLUSION IN SEMI-INFINITE TWO-DIMENSIONAL DOMAINS

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ABSTRACT. We consider an inverse Dirichlet boundary value problem for the Laplace equation that consists of the reconstruction of the shape of a bounded domain contained within a semi-infinite region from Cauchy data observed on a part of the infinite boundary. The numerical solution of this problem is obtained by a hybrid method with the corresponding integral equations of the first kind derived by a Green's function technique. To solve the integral equations we use Tikhonov regularization with sinc and trigonometric quadratures for integrals with various singularities. The numerical examples illustrate the feasibility of the hybrid method in the case of $2D$ semi-infinite regions.

1. Introduction. In nondestructive testing, one tries to assess the interior structure of an object from some information measured on the accessible boundary. Such problems are of particular interest for the case of unbounded domains. The mathematical modeling of thermal or electrostatic imaging methods in nondestructive testing and evaluation leads to inverse boundary value problems for the Laplace equation. In principle, in these applications inclusions or interior cracks are detected from overdetermined Cauchy data on the accessible part of the boundary [**1, 2, 14**].

For simplicity of our presentation we assume that $D_1 \subset \mathbf{R}^2$ is a semi-infinite region with boundary Γ and D_0 is a simply connected bounded domain in $\mathbf{R}^2$ with boundary $\Gamma_0 \in C^2$ such that $\overline{D}_0 \subset D_1$. Further, we denote $D := D_1 \setminus \overline{D}_0$.

Keywords and phrases. Laplace equation, semi-infinite region, inverse boundary value problem, Green's function, integral equation of the first kind, collocation method, trigonometric interpolation, sinc approximation, hybrid method, Tikhonov regularization.
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