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NONLINEAR INTEGRAL EQUATION METHODS FOR THE RECONSTRUCTION OF AN ACOUSTICALLY SOUND-SOFT OBSTACLE

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ABSTRACT. The problem considered is that of determining the shape of a planar acoustically sound-soft obstacle from knowledge of the far-field pattern for one time-harmonic incident field. Two methods, which are based on the solution of a pair of integral equations representing the incoming wave and the far-field pattern, respectively, are proposed and investigated for finding the unknown boundary. Numerical results are included which show that the methods give accurate numerical approximations in relatively few iterations.

1. Introduction. In applications such as medical imaging, nondestructive testing, prospecting for oil and gas, and radar and sonar obstacle detection, a typical problem is to find the shape of an unknown obstacle using information from the influence that the obstacle has on propagating waves. This situation can be modeled mathematically as a so-called inverse obstacle scattering problem. In this paper we consider, for simplicity, an inverse scattering problem where the shape of a planar acoustically sound-soft obstacle is to be determined from measurements of the far field pattern (the asymptotic behavior of the scattered field at large distances from the obstacle) for one time-harmonic incident field with wave number $k > 0$. The ideas presented can, in principle, be applied to other types of inverse scattering problems for time-harmonic waves, e.g., sound hard and impedance boundary conditions. The difficulty found when trying to solve for the shape of an obstacle is due to the fact that the problem is both nonlinear and severely ill-posed in the Hadamard sense [**7**].

Keywords and phrases. Far field pattern, ill-posed, inverse scattering, Newton method, Tikhonov regularization, sound-soft.

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