JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 19, Number 3, Fall 2007

FAST METHODS FOR THREE-DIMENSIONAL INVERSE OBSTACLE SCATTERING PROBLEMS

HELMUT HARBRECHT AND THORSTEN HOHAGE

ABSTRACT. We study the inverse problem to reconstruct the shape of a three dimensional sound-soft obstacle from measurements of scattered acoustic waves. To solve the forward problem we use a wavelet based boundary element method and prove fourth order accuracy both for the evaluation of the forward solution operator and its Fréchet derivative. Moreover, we discuss the characterization and implementation of the adjoint of the Fréchet derivative. For the solution of the inverse problem we use a regularized Newton method. The boundaries are represented by a class of parametrizations, which include non star-shaped domains and which are not uniquely determined by the obstacle. To prevent degeneration of the parametrizations during the Newton iteration, we introduce an additional penalty term. Numerical examples illustrate the performance of our method.

1. Introduction. It is well known that the propagation of an acoustic wave in a homogeneous, isotropic and inviscid fluid is approximately described by a velocity potential $U(\mathbf{x}, t)$ satisfying the wave equation $U_{tt} = c^2 \Delta U$. Here, c denotes the speed of sound, $v = U_{\mathbf{x}}$ is the velocity field and $p = -U_t$ is the pressure. For more details on the physical background, we refer the reader to the monograph [**5**]. If U is time harmonic, that is, $U(\mathbf{x}, t) = \operatorname{Re}(u(\mathbf{x})e^{-i\omega t})$, $\omega > 0$, in complex notation, then the complex valued space-dependent function u satisfies the Helmholtz equation

$$\Delta u + \kappa^2 u = 0 \text{ in } \mathbf{R}^3 \setminus \overline{\Omega}. \tag{1.1}$$

Here, $\Omega \subset \mathbf{R}^3$ describes an obstacle and $\kappa = \omega/c$ is the wave number. We assume that Ω is bounded, that $\mathbf{R}^3 \setminus \Omega$ is simply connected and that the boundary $\Gamma = \partial\Omega$ is smooth. For sound-soft obstacles the pressure p vanishes on Γ, which leads to the Dirichlet boundary condition

$$u = 0 \text{ on } \Gamma. \tag{1.2}$$

Received by the editors on November 23, 2005, and in revised form on September 13, 2006.