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A REVIEW OF LINEAR AND NONLINEAR CAUCHY SINGULAR INTEGRAL AND INTEGRO-DIFFERENTIAL EQUATIONS ARISING IN MECHANICS

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ABSTRACT. This study is primarily concerned with the presentation of a review of a collection (which could be regarded as a "test set") of linear and nonlinear singular integro-differential equations with Cauchy kernels, all of which arise from practical applications in Applied Mathematics and Mathematical Physics. The main objective of this review is to provide numerical analysts and researchers interested in algorithm development with model problems of genuine scientific interest on which to test their algorithms.

Brief details of the methodology of derivation of the equations are provided and, where possible, existence, uniqueness and asymptotic results are discussed. References are also given to other studies that have dealt with similar problems. The importance of carrying out the necessary mathematical analysis is emphasized for one class of problems where it is shown that the solution abruptly ceases to exist as a parameter is varied. It is further shown that developing asymptotic estimates for the behavior of the solutions is very often a crucial component in the design of effective numerical methods. The importance of regularization is discussed for a class of problems, specific conclusions are drawn and recommendations are discussed. An appendix contains further related problems that may be used for further comparison purposes.

1. Introduction. Many practical problems in elasticity, crack theory, wing theory and fluid flow give rise to singular integral equations with Cauchy kernels (see, for example [**3, 37, 61**]). For linear equations, a great deal of theory exists. Indeed, Muskhelishvili in his famous book [**77**] presents a thorough analysis of linear singular integral equations (LSIEs) providing closed form solutions. For this class of problem a considerable number of numerical techniques also exist. Schemes based on Galerkin methods [**39, 44**], collocation methods [**14, 16, 17,**

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