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THE DISCRETE MULTI-PROJECTION METHOD FOR FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. In this paper, discrete multi-projection methods are developed for solving the second kind Fredholm integral equations. We propose a theoretical framework for analysis of the convergence of these methods. The theory is then applied to establish super-convergence results for the corresponding discrete Galerkin method, collocation method and their iterated solutions. Numerical examples are presented to illustrate the theoretical estimates for the error of these methods.

1. Introduction. Let $\mathbf{X}$ be Banach space and $\mathcal{K}$ a compact linear operator from $\mathbf{X}$ to $\mathbf{X}$. For a given $f \in \mathbf{X}$, suppose that we want to find a $u \in \mathbf{X}$ such that

$$(\mathcal{I} - \mathcal{K})u = f. \tag{1.1}$$

Let $\mathbf{N} := \{1, 2, \dots\}$. The projection method for approximately solving (1.1), cf. [**3, 5, 11**], would be the following. First, select a sequence of linear subspaces $\{\mathbf{X}_n \subset \mathbf{X} : n \in \mathbf{N}\}$, and a sequence of projection operators $\{\mathcal{P}_n : \mathbf{X} \to \mathbf{X}_n : n \in \mathbf{N}\}$, then use $\mathcal{K}_n := \mathcal{P}_n \mathcal{K} \mathcal{P}_n$ (or $\mathcal{K}_n := \mathcal{P}_n \mathcal{K}|_{\mathbf{X}_n}$) as an approximation of $\mathcal{K}$, and find $u_n \in \mathbf{X}_n$ such that

$$(\mathcal{I} - \mathcal{K}_n)u_n = \mathcal{P}_n f. \tag{1.2}$$

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