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CONSTANT-SIGN SOLUTIONS OF A SYSTEM OF INTEGRAL EQUATIONS WITH INTEGRABLE SINGULARITIES

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ABSTRACT. We consider the following systems of Fredholm integral equations

$$u_i(t) = \int_0^1 g_i(t,s) f_i(s, u_1(s), u_2(s), \dots, u_n(s))\, ds, \quad t \in [0,1], \quad 1 \le i \le n$$

$$u_i(t) = \int_0^\infty g_i(t,s) f_i(s, u_1(s), u_2(s), \dots, u_n(s))\, ds, \quad t \in [0,\infty), \quad 1 \le i \le n$$

and the system of Volterra integral equations

$$u_i(t) = \int_0^t g_i(t,s) f_i(s, u_1(s), u_2(s), \dots, u_n(s))\, ds, \quad t \in [0,T], \quad 1 \le i \le n,$$

where the nonlinearities f_i, $1 \le i \le n$ may be singular in the independent variable and may also be singular at $u_j = 0$, $j \in \{1, 2, \dots, n\}$. Our aim is to establish criteria such that the above systems have at least one *constant-sign* solution $(u_1, u_2, \dots, u_n)$, i.e., for each $1 \le i \le n$, $\theta_i u_i \ge 0$ where $\theta_i \in \{1, -1\}$ is fixed.

1. Introduction. In this paper we consider three systems of singular integral equations. Specifically we are interested in the following

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