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SUPERCONVERGENCE OF PIECEWISE POLYNOMIAL COLLOCATIONS FOR NONLINEAR WEAKLY SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. The piecewise polynomial collocation method is discussed to solve nonlinear weakly singular integral equations. Using special collocation points, error estimates at the collocation points are derived showing a more rapid convergence than the global uniform convergence in the interval of integration available by piecewise polynomials. For instance, using piecewise linear collocation, the convergence rate at collocation points is $O(h^4)$ if the singularity of the kernel is sufficiently mild (the global convergence rate is $O(h^2)$).

1. Introduction. Numerical methods for linear Fredholm integral equations of the second kind have been studied extensively during the last 20 years. More recently, much of this analysis has been extended to nonlinear integral equations, either to Hammerstein equations or to general Urysohn equations. For a comprehensive description of the literature see, for example, a survey paper by Atkinson [**1**].

Special attention has been paid to collocation methods for solving Hammerstein equations. A new-type collocation method was presented by Kumar and Sloan [**8**] and its superconvergence properties were studied by Kumar [**6**]. The connection between Kumar and Sloan's method and the iterated spline collocation method was discussed by Brunner [**3**]. Two discrete collocation methods were presented by Kumar [**7**] and Atkinson and Flores [**2**]. A spline collocation method and a product integration method for the weakly singular Hammerstein equation were studied by Kaneko, Noren and Xu [**4**].

Numerical methods for multidimensional weakly singular integral equations were studied by Vainikko [**14**]. Global convergence estimates have been derived, whereas the superconvergence effect at collocation points has been analyzed only for linear equations.

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