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# DISCRETE COLLOCATION FOR A FIRST KIND CAUCHY SINGULAR INTEGRAL EQUATION WITH WEAKLY SINGULAR SOLUTION

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ABSTRACT. A fully discrete scheme for the numerical solution of a first kind Cauchy singular integral equation is analyzed. The underlying mesh may be graded in order to approximate weakly singular solutions (functions behaving like $|t|^\alpha$, $\alpha > 0$, $t \in [-1,1]$) as well as smooth ones on a uniform grid. Order of convergence results are established in sup-norms and weighted $l_2$-norms. They exactly reflect the outcome of the numerical computations. For the stability analyses it is shown that the row differencing imbedded in the method just yields a Moore-Penrose inverse in an unconventional way.

**1. Introduction.** This paper concerns a fully discrete numerical method for the solution of Cauchy singular integral equations on a smooth closed curve. The method was introduced in [**4, 11**] and is closely connected to the midpoint collocation studied in [**3**]. These papers provided a first step towards the analysis of nonuniform meshes which are useful for nonsmooth solutions. Obviously, adaptive mesh refinement will produce such grids for rapidly varying or weakly singular solutions. But the theory did not yet cover completely these cases and did not reflect comprehensively the order of convergence achieved with the methods in practice. Here we will take a second step by giving optimal error estimates even for weakly singular solutions approximated on an appropriately graded mesh. We study in detail the equation

$$\mathcal{H}u(t) = f(t), \quad t \in [0, 2\pi], \tag{1.1}$$

with the Hilbert transform on the unit circle (cf. [**6**], e.g.) which is defined for Hölder continuous functions $u \in C^\alpha[0, 2\pi]$, $\alpha > 0$, as

$$\mathcal{H}u(t) := \frac{1}{2\pi} \int_0^{2\pi} u(s) \cot \frac{t-s}{2} \, ds, \quad t \in [0, 2\pi].$$

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